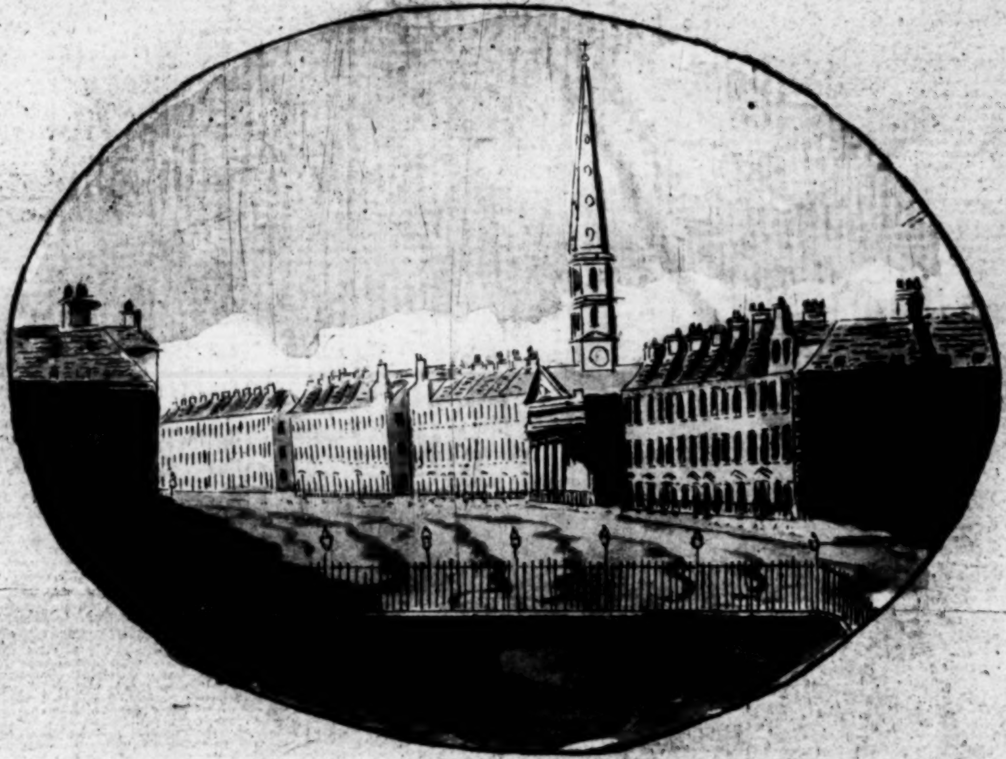

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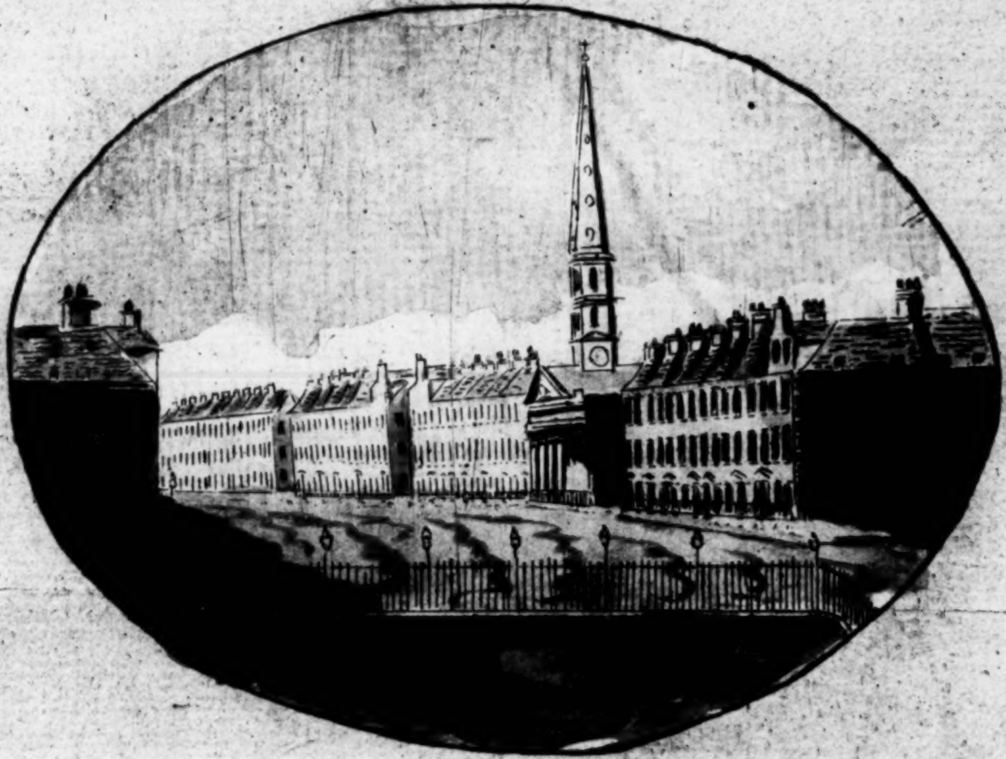

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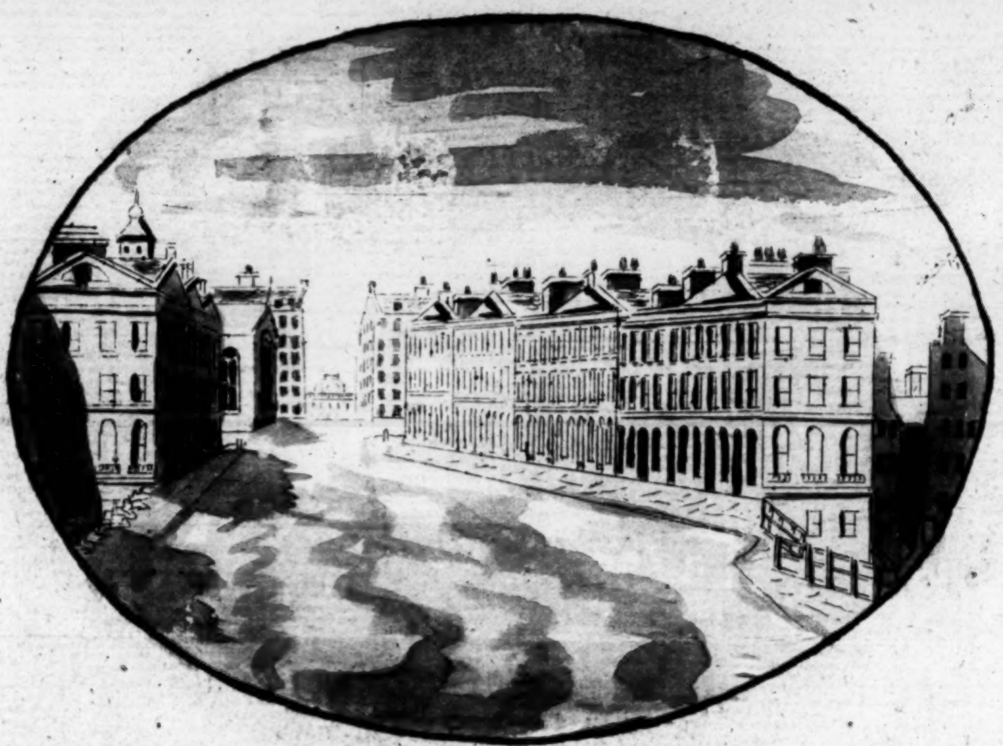

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B. HENNIKER

THE
Principles of Perſpective,

EXPLAINED IN

A GENUINE THEORY,

AND APPLIED IN

AN EXTENSIVE PRACTICE;

WITH THE

CONSTRUCTION AND USES OF ALL SUCH
INSTRUMENTS AS ARE SUBSERVIENT
TO THE PURPOSES OF
THIS SCIENCE.

By ROBERT BRADBERRY.

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PREFACE.

NO mathematical ſcience requires a *Theory* more than *Perſpective*, and perhaps no one has been leſs attended with it in publications of this kind. The complex manner in which moſt authors have treated the theoretical part has not been ſatisfactory to me; and as many others are of my way of thinking, I here preſent them with a *Theory of Perſpective*, which appears to me the moſt genuine, natural, and perſpicuous that poſſibly can be; as it conſiſts of the feweſt, and thoſe the moſt ſimple principles, viz. the motion of a point in a given direction, and of a viſual ray proceeding from it to the eye at a given diſtance; which ray, by its interſection

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with the perſpective plane, marks out the perſpective appearance of the point or object upon it.

The application of this theory to practice, in the common way, is ſo eaſy and natural, that I think what is here ſaid may ſuffice for that purpoſe; but, as there are many important particulars relative to the *Theory* and *Practice* of *Perſpective* not touched upon in treatiſes of this ſort, I have added as much on that head as I judged was neceſſary to give the ſtudent of the *Polite Arts* as juſt a notion of this ſcience as it deſerves, and as he ought to conceive of it.

Genius and inclination for a particular ſtudy may, by labour and aſſiduity, ſurmount many difficulties; yet to remove impediments from the path of ſcience is a grateful taſk, and genius will feel and acknowledge the obligation; for thoſe who are moſt capable of profiting by inſtruction are generally the moſt ſenſible of its value.

I HAVE purpoſely avoided all kind of prolixity, and ſtudied one point only here, viz. to make the way to ſcience as eaſy and conciſe as poſſible, and with the leaſt expence of money and time. The *Patronage* I have received from a *Generous Public*, (during my exhibiting Philoſophical and Optical Effects, &c.) I ſhall ever with gratitude acknowledge; and purpoſe, on ſome future day, (at the requeſt of thouſands) to publiſh a *Treatiſe* on *Optics*, in which ſhall be laid down the moſt certain methods of producing every wonderful and beautiful effect that has hitherto appeared in *Optics*, and ſome that never have yet been thought of by any perſon but myſelf; Subſcriptions for which I take in at Mr. Slapp's, the corner of Furnivall's Inn Court, Holborn, London; but as it will be ſome time before the work can be brought before the Public, no money will be taken on ſubſcribing, nor till the work be ready for publiſhing.

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I. The GENUINE THEORY of PERSPECTIVE, demonſtrated from its Firſt Principles.

PERSPECTIVE is the art of delineating upon a plain ſurface the appearance of objects, ſuch as they have upon a plate of glaſs placed between the eye and any object we look at, when the eye is placed at a given diſtance from the glaſs.

For ſince the ſenſation, or viſion of any point is cauſed by a *ray of light* proceeding from that point to the optic nerve, or *retina*, in the eye, and as this ray paſſes through the tranſparent plane, or plate of glaſs, in its way to the eye, the point in which it tranſits the plane is that which we call the *place*, or *ſeat*, of that point upon the *perſpective plane.*

But this doctrine of the *perſpective ſeat* of a *point*, a *line*, a *ſuperficies*, and a *ſolid*, muſt be illuſtrated by proper diagrams, that it may become eaſy and perſpicuous; for that method can only convey *clear ideas* that is in itſelf *natural* and *ſimple*; and ſuch a one I ſhall here endeavour to purſue.

This method conſiſts in the conſtruction of a *perſpective priſm*, and its interſection by the *perſpective plane.* The *priſm* is repreſented by YGHIKLMN, and the *perſpective plane* by ABCD. The *interſection* of the *priſm* and *plane* is the triangular plane O*y*Q (Fig. 1.)

The baſe of the priſm IGMK is the *ground plane*, in which all objects in the *plane of the horizon* are conſidered, and their perſpective ſeats determined in the plane AC. The vertex Y of the priſm is the place, or poſition of the eye at its given diſtance Y*y* from the perſpective plane, which is here ſuppoſed to be poſited directly before the eye, or at right angles to the priſm.

THE part of the plane AD, which ſtands on the ground, is called the *ground line*; the point *(y)* is called the *point of ſight*; and the line E*y*F (parallel to AD) the *horizontal line.*

THEN in order to determine the perſpective appearance of objects upon the ground, or horizontal plane IGMK, indefinitely continued out towards *cd*, we muſt firſt conceive a point to begin to move from the plane AC along the indefinite right line OD, which is perpendicular to the ground line AD; this point is at the ſame time in the ſide IN of the priſm, and drawing the dotted line YO, that will repreſent the *viſual ray*, by which the ſaid point is ſeen by the eye at Y.

Let the point in its motion along the line OD arrive at the ſituation T, and draw the viſual ray TY by which it there appears. Then it is ſelf-evident that the viſual rays YO, YT are in the plane, or ſide of the priſm IN; alſo that the ray YT paſſes through the perſpective plane in the point *(t)*, which therefore is the *perſpective ſeat* of the point T.

As the motion of the point is continued, the angle TYN conſtantly decreaſes, till at length the point gets to an indefinite diſtance, and then that angle vaniſhes, and the ray

YT coincides with the line YN, and the object now in the *horizon* has its perſpective ſeat on the plane AC in *(y)*.

By this indefinite motion from the perſpective plane in O to the horizon, the viſual ray YT keeps conſtantly moving in the ſide or plane of the priſm IN, and of courſe it muſt trace out, by its paſſage through the perſpective plane, the line O*y*, which is the common interſection of the two planes IN and AC; and therefore the ſaid line O*y* is the *perſpective ſeat*, or appearance of the line, or ſide O*d* of the ground plane indefinitely extended.

In like manner, if a point moves from Q along the line Q*c* indefinitely extended, the viſiual ray will deſcribe the line Q *y* on the perſpective plane in which it interſects the ſide GN of the priſm; alſo another point moving from P in the line P*e* to an infinite diſtance, the ray proceeding from it will deſcribe upon the plane AC the line P*y*, which is the common ſection of that plane, with the vertical plane of the priſm HN.

Hence, it is again ſelf-evident, that if a line QO were to move from the ground line to an indefinite diſtance, in a direction O*d*, and always parallel to itſelf, or to the ground line, it would appear on the perſpective plane to deſcribe the triangle O*y*Q, which therefore will be the perſpective of that indefinitely extended horizontal plane *d*OQ*c*, deſcribed by ſuch a motion.

Hence, alſo, it is manifeſt that any line QO, however large at the perſpective plane, yet removed to an indefinite diſtance, viz. to the *horizon*, will there appear in the line

EF, paſſing through the point *(y)* which is therefore called the *horizontal line.*

All indefinite right lines perpendicular to the ground line of the plane, as O*d*, P*e*, Q*c*, have their *perſpective* expreſſed by right lines converging to the point of ſight *(y)*, as O*y*, P*y*, Q*y*, which are called *radial lines.*

The *perſpective* of any determinate ſpace QOTR is thus found on the plane; *t* and *r* being the *ſeats* of the points T and R on the ground plane, O*t* and Q*r* will be the perſpectives of the ſides OT and QR; and drawing *rt*, that muſt be the perſpective of the ſide RT; and ſo the *trapezium* QO *tr* will be the perſpective of the propoſed area QOTR.

If OT be equal to QR, then is O*t* equal to Q*r*, and *tr* is parallel to OQ. If OT=QO, then is the area OTRQ a ſquare, and O*tr*Q the perſpective of that ſquare.

Let the diagonal OR of the ſaid ſquare be continued out indefinitely towards *f*; then upon the horizontal line EF, take *y*X and *y*Z equal *y*Y, the diſtance of the eye from the perſpective plane, and draw XY. Alſo make O*b*=OI (=Y*y*) and draw *ba* parallel to QO or RT; and O*ba* being a right angle, O*a* will be the diagonal of a ſquare, and equal and parallel to YX.

Then if a point be now conceived to move along this diagonal line O*f*, to an indefinite diſtance, the *viſual ray* will contain an angle with the line YX that will conſtantly decreaſe, and at laſt vaniſh, when the point is in the horizon; and at laſt, by a coincidence of the ray with the line YX, the ſaid point in the horizon will have its perſpective ſeat at X in the horizontal line EF. During this motion of the point in the diagonal of a ſquare, the viſual ray will trace out the line OX in the perſpective plane. Wherefore, becauſe this line O*f*, and all others parallel to it, converge to the point of diſtance X upon the plane, they are called *diagonal lines.*

Hence any indefinite line drawn from the point O between OT and OR, will have its perſpective ſeat in ſome part between O*y* and OX, and every ſuch line between OR and OQ will have its ſeat in the ſame horizontal line beyond X; and ſuch points in the line EF to which theſe lines tend, are called *accidental points.*

Hence, the line X*y*, between the point of ſight *y*, and point of diſtance X, is the perſpective of 45 degrees in the horizon, or the double thereof XZ, is the perſpective of 90 degrees in the horizon.

HENCE alſo it is evident, that ſince the original lines Q*c* and O*f* croſs each other in R, therefore the ſeat of the point R will, in the perſpective plane, be at (*r*) where the radial Q *y* is interſected by the diagonal OX. And from thence the following General Rule is derived for finding the perſpective ſeat of any given point or object upon the ground plane, *viz.*

GENERAL RULE.

FROM the given point, as T, draw a perpendicular to the ground line, as TO; from the point O ſet off the diſtance OT from O to Q in the ground line, and draw the diagonal QZ, and where it croſſes the radial, as at (*t*), there will be the ſeat of the point T required.

II. A DEMONSTRATION of the foregoing GENERAL RULE of PRACTICAL PERSPECTIVE, in Regard both to Direct and Oblique Views.

THE *theory* of *perſpective* required a poſition of the *perſpective plane* ABCD (Fig. 1.) with an elevation above the *ground plane*, which in that figure, as in moſt real caſes, is at right angles to it. But the practical rule we have thence deduced, admits a *deliniation* in *perſpective* of any ſort of objects with the utmoſt truth and facility, by placing the *perſpective plane* upon the *ground* or *horizontal plane*.

For inſtance, (Fig. 2.) let ABCD be the

tranſparent or perſpective plane in an horizontal poſition, and let T be the given point whoſe perſpective ſeat is to be found on the plane AC in this poſition; and then by the rule the proceſs is the ſame here as before in the other figure.

For drawing the perpendicular TO to the ground line AB, and from O the radial O*y* to the point of ſight *(y)*; and then making QO=OT, and drawing the diagonal QZ to the point of diſtance Z, it will interſect the radial O*y* in the point *(T)* the *perſpective ſeat* of the point T as required; therefore alſo *OT* will be the perſpective of the line OT.

To demonſtrate the truth of this method, it will be neceſſary to make the following conſtruction in the diagrams, viz. Take OV in the ground line (in both figures) equal to the diſtance of the eye OI (=yZ) and draw ZV, which will be parallel to the radial YO. Then (in Fig. 1) there are two ſimilar triangles formed, viz. ITY, OTt; and VQZ, OQT (in Fig. 2.)

From the firſt we have this analogy, IT : OT : : IY : Ot; and from the ſecond this QV : QO : : ZV : OT. Now the three firſt terms in each analogy are reſpectively equal to each other, that is IT=QV, OT=OQ, and IY (=Oy)=ZV. Therefore the fourth term Ot *is the ſame in the firſt* with OT in the ſecond; conſequently the interſection of the radial Oy and diagonal QZ is the ſeat of the point T as true by the *practical method* in Fig. 2. as by the theory in Fig. 1.

If, therefore, OTRQ be the ſquare before mentioned, its perſpective will be O$Tr$Q as determined by the theory, and OR and QT will be ſeated in O*r* and Q*T*: the centre N of the ſquare in *(n)* and the ſides QR and RT in Q*r* and *r*T.

FROM what has been ſaid, it will be eaſy to conceive how the perſpective of any figure may be put into perſpective by inſcribing it in a ſquare, and in the perſpective of that ſquare marking the ſeat of all the points in the extreme parts or angles of the given figure; and then laſtly, by connecting theſe points with right lines, the perſpective of the given figure is formed.

THUS, for example, (in Fig. 3.) let ABCD be the ſquare upon the ground plane, placed directly before the eye at *(y)*; in this ſquare are inſcribed the following figures, (Fig. 1.) The ſquare EOFQ with an angle F in front; (2.) A circle circumſcribing that ſquare; (3.) A ſquare inſcribed in this circle, in a ſituation ſimilar to the orginal ſquare; (4.) Another inſcribed circle, and in that, (5.) An iſoſceles triangle ZVX; (6.) The ſmall ſquares IR, &c. in the angles of the original ſquare.

Then it is plain, by inſpection only, how all the ſeats of the points, O, I, B, R, E, &c. are found in the perſpective ſquare A*bc*D; and by properly connecting them, how all the perſpectives of the ſeveral figures reſult; and even circles are eaſily traced with an even and dexterous hand through ſo many given points. Theſe figures are readily ſeen and diſtinguiſhed by the correſponding ſmall letters in each.

The forms and ſhapes of figures in their perſpective plane are greatly altered in a *direct view*, but ſtill more ſo in one that is *oblique*, as is evident (Fig. 4.) where the ſame ſquare is placed on one ſide the point of ſight *(y)*, and its perſpective A*bc*D determined by the radials A*y*, D*y*, and the diagonal DX contains the perſpective of the circle and ſquare in front, very different in their forms, dimenſions, &c. from what they had before in direct view.

If the original ſquare be divided by equidiſtant lines, perpendicular to the ground line AD, as 11, 22, 33, &c. their perſpectives all converge to the point of ſight *(y)*, and the perſpective line *bc* will be equally divided by them. Any lines, GH, OQ, IK, parallel to the ground line by transferring their diſtances to the ground line, (that is, making D1=DH, D2=DQ, &c.) and then drawing the diagonals X1, X2, &c. the points *h*, *q*, *k*, *c*, will be given in the radial D*y*, through which to draw their perſpectives, *g*, *h*, *oq*, *ik*, *bc*, in this *oblique view*, the ſame as in the *direct* one.

Though right lines, perpendicular or *oblique* to the ground line, differ in their dimenſions in the direct and oblique views, yet thoſe right lines that are *parallel* to the ground line are the ſame in both views. For, in the *oblique view* we have AD : *bc* : : *y*D : *yc*; and in the direct view it is AD : *bc* : : *y*A : *by*; but in the former *y*D is to *yc*, in the ſame ratio as *y*A to *yb* in the latter; therefore the ratio of AD to *bc* is the ſame in both, conſequently if AD be the ſame in both, then will *bc* be ſo too.

III. The PRINCIPLES of SCENOGRAPHIC PERSPECTIVE explained and demonſtrated.

HAVING demonſtrated the perſpective principles of *Ichnography*, or all that is concerned in the *ground plane*, we have in effect done the ſame with reſpect to *Scenography*, or all that relates to *elevations*, or *upright ſtructures*, built upon the ſaid ground plane.

FOR whether any right line, as AD, (Fig. 5.) be upon the ground, or elevated above it, as AB, yet if thoſe lines are at an equal diſtance A*y*, from the point of ſight, their perſpective diminutions will be equal; and they equally reſult from, and may be determined by the ſame general rule before mentioned; for draw the radials A*y* and D*y*, and the diagonal AZ, which will croſs the radial D*y*, in (*d*); and draw *a*, *d*, parallel to AD, then will this line *a d* be the perſpective of a line equal to AD, and parallel to it, and at the ſame diſtance from it. Conſequently A*a d*D will be the perſpective of a ſquare upon the ground plane whoſe ſide is AD.

In like manner, upon the point D erect the perpendicular DC=AD, and compleat the ſquare ABCD, then draw the radials B*y*, C*y*, and the diagonal BZ, cutting C*y* in (*c*) and draw *bc* paralled to BC; ſo ſhall B*b c*C be the perſpective of the ſame ſquare, viz. equal to ABCD placed upon the ground plane parallel to AD, and elevated above it to the height of AB, and at the ſame diſtance from it.

By joining the points *a*, *b*, and *c*, *d*, 'tis evident the whole figures become the perſpective of the cube ſtanding on the ground line AD, whoſe ſide in front, or next the eye, is ABCD, the baſe A*a* *d*D, the top B*b* *c*C, the right ſide A*a* *b*B, the left ſide D*d* *c*C; and the fartheſt or hinder ſide *ab* *cd*.

What remains is to ſhew that the perſpective *ab* of a given line, in an elevation above the ground plane, is equal to the perſpective of it *ad*, when it coincides with the ſame; and this is eaſily ſeen in the ſimilar triangles A*y*D, *ayd*, and A*y*B, *ayb*. For from them we have theſe ratios AD : *ad* : : A*y* : : AB : *ab*; but AB=AD, therefore *ab*=*ad*. And this holds good in every elevation of the line AB, ſince the angle BAD is not concerned in the above analogies.

If the ſides of the cube be continued up to the line of ſight *y*Z, there will be formed an equal-ſided parallelopiped upon the ſame ſquare baſe A*a* *d*D, but whoſe upper ſurface vaniſhes into a right line *e*E*f*F, the front view of this ſolid is AEFD; the ſide A*a* *e*E is called the profile or ſide-view, which is greater or leſs in proportion to the obliquity of the view; for in front, or a direct view, it vaniſhes, as is eaſy to obſerve from the reaſon of the thing in Fig. 3. and 4.

If the parallelopiped were carried to a height above the line of ſight equal to CF, and was hollow or tranſparent, then would the under part of the top or higheſt ſurface have its perſpective exactly equal to B*bc*C; and if it extended equally above and below the line *y*Z, then the perſpective of the top will be the ſame with that of the bottom, viz. A*a* *d*D.

When it is conſidered, that by having the diſtance, altitude, angle of poſition, and profile of any object given, the perſpective ſeat of every point, and therefore of every line, will be eaſily formed by the foregoing Practical Rule, it will be needleſs to multiply words to ſhew how the perſpective of any object may be compleatly found by delineating thoſe of the points and lines which compoſe it, as they lie in the perſpective ſides of the cube or parallelopiped, which circumſcribe the object, as the *point of ſight* and the point of diſtance are always known, and alſo the accidental points from the given poſition of the object with reſpect to the ground line, there can remain no difficulty to the ingenious artiſt.

And hence, that which has been hitherto conſidered as a mighty difficulty in perſpective, viz. drawing the perſpective of objects by reflection, as thoſe ſeen in water, &c. will be found none at all. For if they are things in the heavens, their altitude meaſured by a quadrant gives their elevation above the horizontal line HZ; and ſince the angle of the incident ray is ever equal to that of the reflected one, therefore the point of the image is in a right line drawn from the object perpendicular to the horizon HZ, and at the ſame diſtance below as the object itſelf is above it. Hence the place of the ſun, clouds, tops of mountains, &c. in day pieces; and the moon, ſtars, &c. in night views, may be readily and truly drawn in perſpective, by the ſame general rule.

But if the objective OB be upon the ground, (Fig. 11.) and its height and diſtance AB be known, then divide the diſtance AB in the point D, ſo that it may be BD : AD : : OB : AC=to the height of the eye at C, alſo make IB=OB and draw CI, then will the angle CDA=IDB=ODB; and conſequently if water be at D, the image at I will be ſeen by the eye at C. Or a general rule for finding AD is $\frac{AB \times AC}{OB + AC}$=AD. But more generally and conciſely by Gunter's lines proceed thus: Let the altitude of any body, on the earth or in the heavens, be denoted by the angle ADC, which ſuppoſe to be 30 degrees, and let the height of the eye AC be ſix feet. Then if you extend the compaſſes from 30 to 45 degrees in the line of tangents, that extent will reach from ſix feet to 10,4 feet in the line of numbers, which will be equal to AD, the diſtance of the point D in the plane of reflection, or azimuth of the phænomenon, from the eye. The diſtance AD being found in the ground plane, its ſeat on the perſpective plane, or picture, will be found by the interſection of a radial and diagonal as above ſhewn.

IV. The ANALYSIS of PERSPECTIVE; or the Method of Analyſing a given Perſpective View or Landſcape.

THE *analyſis* of *perſpective* conſiſts in the reſolution of a *perſpective picture* into its *prototype*, or original component parts, ſo that we may form a juſt idea of their ſituations, diſtances, dimenſions, &c. from their delineation in a given *landſcape*, or perſpective picture. The firſt ſtep to be taken in this proceſs, is to conſtruct two *lines of meaſures*, one for taking the dimenſions, diſtances, &c. of objects upon the *ground plane*, and the other for thoſe which have an elevation above it. The firſt of theſe is to be laid upon the ground line, as AG Fig. 5. and is nothing more than the diviſion of the loweſt line of the landſcape, or picture into *equal parts*, which may repreſent *feet*, *yards*, or *miles*, as the nature of the perſpective require. The ſecond *line* of *meaſures* for uprights, is to be affixed to the perpendicular line GH on the ſide of the landſcape. By the line of meaſures on the ground line AG, the length of any perſpective line parallel thereto may be inſtantly known; as alſo the height of any line perpendicular to the perſpective plane, or landſcape, in any part, may be eſtimated by the line of meaſures at the ſide.

In this operation we have regard to the point of ſight, *(y)*; and this in any landſcape is eaſily found, by continuing out any two extreme radial lines, as AN and GM, till they meet, which will be in the point *(y)* required. In the diagram, Fig. 5. the ſcale of meaſure upon the ground line AG contains fifteen equal parts, *feet*, *yards*, &c. and the perpendicular ſcale GH has eight. And, therefore, if lines were drawn by the edge of a rule from the point *(y)* to each of thoſe diviſions, they would ſhew in any part of the ground plane, or ſide plane, the proper length of any perſpective lines parallel to the ſaid ſcales, and of courſe the form of *walks*, *canals*, *viſtas*, &c. which are perpendicular to the ſaid ſcales. Thus, for example, ſuppoſe a ruler laid from *(y)* to 2, and by its edge you draw the line *n*2; then will the ſpace AN *n*2 be the perſpective of a walk or viſta of two yards in width, by the ſide of the *perſpective garden*. And for the ſame reaſon GM*m*13 will be ſuch another on the other ſide; and 7*r*R9 will be the ſame through the middle of the area.

But now to reſolve and meaſure any *parallelogram* in perſpective parallel to the ground line, will require firſt a determination of the point of diſtance Z; for by *diagonal lines* drawn from the diviſions of the ſcale AG to that point, the extreme line AN in the ſide of the landſcape is divided, and by theſe diviſions the perſpective diſtances of objects between any two parallels to the front line AG are eſtimated or become known. Thus, if the diagonal z2 be drawn, it will croſs the line AN in *t*, and drawing through *t* the parallel T*t*, it will make the perſpective walk TA of two yards width, contiguous to the ground line. Again, if diagonals from Z to 7 and 9 be drawn, they give two points in the radial AN at Q, thro' which two parallels are drawn, which form the ſame walk QS through the midſt of the garden, and in like manner ſuch a walk is determined at the fartheſt ſide MN. But it is evident, the nearer Z is to *y* the greater will be the diviſions A*t*, AQ, AN; and leſs as it is farther removed. Therefore it muſt be determined and fixed for the given landſcape, which is not always very eaſy to be done. But no difficulty can ariſe in ſuch perſpective pieces as have any regular figure in a ſide view upon the ground line.

To analize a building, or any other object, in any part of the landſcape, you are to obſerve, that all the ichnography of it is reſolved and meaſured by the method juſt mentioned; and the ſcenography is determined by meaſuring the upright lines thus. Let the parallels be drawn for the angular points, and where they interſect the ſide radial, as ſuppoſe at S, there erect perpendiculars equal to the heights on thoſe angles of the buildings as S*s*, then the radial *y*K drawn thro' *s*, will give the meaſure GK of the true or real height of the building in that part, and thus it will be found for any other. Hence the height of trees, houſes, men, &c. are eaſily found on any part of the landſcape, and the manner or ratio of their diminution GK, Sſ, ML, as they are removed from the perſpective plane to the fartheſt part of the view.

HENCE alſo it appears, that objects below the eye riſe conſtantly in the perſpective view towards the horizontal line HZ, and thoſe above the eye in the ſame manner deſcend to it, as they go more remote from the plane. Thus birds diminiſh to our view, and apparently deſcend towards the horizon, when they fly from us in a direction parallel thereto.

V. The MECHANISM of PERSPECTIVE ; or, MECHANICAL CONSTRUCTION of the THEORY thereof, for a Demonſtration to the Sight.

THE mechaniſm of perſpective is the next thing neceſſary for the young deſigner to be acquainted with, and to have often before his eyes. It conſiſts in conſtructing, or building up the theory of perſpective into a real or material form, that the rationale of the ſcience may, as it were, become the object of ſight, and thereby eaſier conceived by the mind.

In order to this, it will be neceſſary to have a rectangular piece of wood (mahogany) about twenty-four inches long, and ſix or eight wide; in this, at about ten inches from the end, is to be made a dove-tail grove, half an inch deep, and wide; and then a frame muſt be made to move or ſlide freely in the ſaid grove, yet ſo as to be always upright or perpendicular to the ſurface on which it ſtands. This baſis or long parallelogram of wood will then anſwer to the general ground plane GHIKLM (in Fig. 1.) The frame to ABCD, the perſpective plane, and the grove to the ground line OQ. At or near the end muſt be placed a ſtile, or wire, repreſented by the perpendicular HY, with a ſmall hole in the upper part at Y, alſo another is to be placed at the other end, denoted by LN, with a perforation at N.

In the frame AC muſt be fixed a large piece of tranſparent horn, ſuch as is uſed for lanthorns; then having a ſquare made of a thin piece of braſs repreſented by QOTR, with fine holes in the angles and ſides, at Q, O, P, R, S, T, and the other parts when neceſſary, the perſpective of the ſquare is to be drawn upon the plate of horn, with a ſharp ſteel point, in the manner ſhewn in Fig. 2.

This being done through the ſeveral points in the perſpective ſquare, as at Q, P, O, T, *s*, *r*, *g*, *b*, and *y*, let ſmall holes be drilled, and then, the horn being fixed in the frame AC, that muſt be ſo placed in the grove GO, that the ground lines of both the original and perſpective plane may coincide as near as poſſible.

Then for the viſual rays, they will be very properly repreſented by fine threads fixed in, and paſſing through the holes in the ſtiles and planes, &c. Thus if upon the board GK be drawn the parallel lines GM, IK, upon paper, paſteboard, ivory, &c. as alſo the lines GI and MK: and alſo if threads fixed in G and M be put through the holes in the ſtiles at Y and N, and then carried down and fixed in the points I and K; and laſtly, if another thread paſs from Y through the point *(y)* in the horn, and be fixed in the hole of N, then will the perſpective priſm be properly conſtructed.

If the line OR be continued out upon the board to *(a)* and a thread be fixed in that point, and from thence carried through a hole in the point of diſtance X in the horn to the point Y in the ſtile, and there fixed, you will have the plane O*a* XY conſtructed, for the demonſtration of what relates to the perſpective of diagonal lines and their parallels.

LASTLY, let threads befix ed in the holes of the ſquare ORQT, and carried through the holes in the plate of horn, and all of them fixed tight in the hole Y of the ſtile HY, then will the ſquare pencil the viſual rays be properly formed for ſhewing the reaſon of every thing which concerns the perſpective of radial lines, or thoſe which are perpendicular and parallel to the ground line; eſpecially, if to theſe are added other threads from holes in the line OT, continued out, and paſſing through correſponding holes in the horn, which will determine the perſpectives or landſcapes of any given parallelogram.

All that has been hitherto ſaid relates to the mechanical conſtruction of the theory; but the practical part of perſpective may alſo be proved and confirmed by experiments made in the ſame machine, with a little variation; for in this caſe there will be no need of threads at all; and inſtead of a ſtile HY in form of a wire, it ſhould have a round plate of braſs on the top, with a ſmall hole in the middle, to look through, like a common ſight vane to quadrants, &c. Then if ſeveral pieces of horn (very ſmooth and tranſparent) were fitted to the frame AC, ſo as to be taken in and out at pleaſure, and upon theſe ſeveral perſpectives of objects drawn, as of a chair, table, inſide of a room, doors open, &c. Then if theſe are ſucceſſively and oppoſitely placed by the ground plane of the original, and viewed through the ſight vane at Y, the prototype and its perſpective will be ſeen perfectly to coincide upon the plate of horn in every point and part; and conſequently the truth of the draught is eſtabliſhed.

In this laſt experiment, inſtead of horn, a plate of glaſs, properly prepared, by gently rubbing it over, ſo as to grey it with emery, and then ſlightly covered with a little ſweet oil, by this means the glaſs will be perfectly tranſparent, and will take the mark of a black lead pencil.

It will be very eaſy to underſtand that the theory of perſpective may be conſtructed for perſpective plane that has any given inclination to the ground plane; ſince the diſtance of the eye from the plane is, in this caſe, *equal to the perpendicular diſtance from the ground line and co-tangent of the plane's* *inclination added together, the radius being the height of the eye*; and all other parts of the proceſs will be the ſame, or ſimilar to thoſe above deſcribed.

VI. The CONSTRUCTION and USE of a PERSPECTIVE TABLE for readily drawing in true Perſpective any natural Scene, or View of Objects propoſed.

THE method of putting any objects into a perſpective view by practical rules derived from the theory, as above delivered and explained, requires the knowledge of their various dimenſions and diſtances, and is a work of time, with many other circumſtances, the conſideration of all which together has made it neceſſary to contrive ſtill more practical and expedient methods of delineating any given object, or group of objects, in true perſpective.

And this has been found as eaſy and practicable by the invention of a perſpective plain table, as can be derived; and of theſe tables there have been many forms and contrivances ſince that firſt invented by Sir Chriſtopher Wren. They differ only in form and conſtruction; the principle of the operation is the ſame in them all. The one which I have here given in a diagram admits, I think, of the greateſt facility and certainty of motion, which is the utmoſt excellency of this ſort of inſtruments. This perſpective drawing table is repreſented in Fig. 8. It may conſiſt of a board, with a rectangular or frame ABCD, to take off and put on upon occaſion, whoſe uſe is to hold firmly a ſmooth ſheet of paper EFGH upon the body or plane of the table, in the ſame manner as in the plain table for ſurveying. But this perſpective table may be made entire, without ſuch a frame, with proper ſprings on the ſides, to hold the paper for the draught.

This table, placed upon a foot and pillar ST, may be put into a *vertical poſition;* it is not neceſſary it ſhould be perfectly perpendicular, and therefore requires no plumb line or ſpirit level. The table being fixed pretty ſteady, you next apply, or fix to it, a ſet of three moveable quadrangles ABIK, IKLM, LMNO, which have all a perfect freedom of motion at their reſpective angles IK, L, M, N, O. Theſe are connected with the lower part of the table by the ends A, B, going into proper groves or holes, with ſpringcatches for holding them very faſt. In order to make them as light as poſſible, it will be ſufficient to have the whole mahogany, except the four ſides IM, KL, LN, NO, which may be ſmall ſlips of braſs. When things are thus diſpoſed, as in the diagram they appear, it is plain the part NO of the upper quadrangle is freely moveable over all the plain of the table, and therefore in the middle at P is ſcrewed faſt on a ſtile, or perpendicular piece PQ, with a point at Q, called the index of the ſtile or tracer.

THIS tracer PQ has in its lower part at P a pencil fixed in the ſcrew, by which it is faſtened to the piece or bar NO. Now, let *(abcd)* be an imaginary tranſparent plane, as in the theory (Fig. 1. and 2.) upon which let us ſuppoſe the perſpective of a houſe, trees, walls, &c. to be delineated; and that this plane is placed upon the upper part of the table CD.

Then it is evident, if the hand holding the pencil at P were to move in ſuch manner as that the index Q might be made to paſs along or deſcribe any line on the perſpective part the pencil itſelf muſt neceſſarily deſcribe a line of the ſame length, and in the ſame poſition upon the paper on the table; and conſequently by this eaſy artifice, all the contours, or outlines of the landſcape will be readily delineated upon the paper on the perſpective table.

Again, it is evident, that the eye, placed at ſuch a height and diſtance from the plane (*abcd*) as belong to the landſcape drawn upon it, will ſee all the lineaments of objects in the ſaid picture coincide exactly with thoſe of their prototypes, or originals, by the viſual rays paſſing to the eye, from correſponding points in each. Therefore it will follow, that if the ſaid perſpective table *(abcd)* were removed, and a ſight-vane fixed in the proper point of ſight, an eye looking through it would perceive the index Q paſs over the ſame lines in the original objects as before in the landſcape, if moved in the ſame manner; and conſequently the pencil at P will deſcribe the ſame lines as before; and thus a true perſpective draught upon the table ABCD will be made of any diſtant objects propoſed.

THIS practice is univerſal for all objects, remote or near; for the ſight-vane being moveable, may be fixed at any diſtance from the table, and height above the horizon, and conſequently may be thereby adapted to any diſtance of objects, or ſize of the drawing propoſed; for from a bare view of Fig. 2. it is evident *that the perſpective plane QQtr, or landſcape will be larger, as the height of the eye* P *y*, *is greater at the ſame diſtance* *y*X.

ALSO, *it is as evident, that for the ſame height of the eye* yP, *the landſcape will increaſe, or decreaſe as the diſtance of the eye from the perſpective plane decreaſes or increaſes.* Hence, when the eye is at a great height, and the diſtance of the plane or table from it very ſmall, the perſpective of a given ſquare will be *enormouſly great*, and almoſt of a *triangular form.* Therefore a *face*, *head*, &c. drawn in the ſquare will be greatly *deformed* when delineated on the perſpective draught of the table. But this very deformed image will appear to an eye at that given height and diſtance as famous and perfect as the original. And thus the whole doctrine of the *anamorphoſis*, or *deformation of images*, is illucidated, as it were, by inſpection.

The doctrine of ſhadows is reckoned a part of perſpective; but it is rather only the reverſe of the direct method; for if (in Fig. 1.) we ſuppoſe a candle placed at Y, at the diſtance HP from the perſpective plane AC, and if OQ*rt* repreſent an *opake object* upon that plane, then it is evident that rays of light proceeding from the candle will project the ſhadow of that object into a perfect *ſquare from* OQRT on the ground plane GK; ſo that no new principles are required for a further proſecution of this ſubject. Another method for drawing the outlines of the objects in perſpective may readily be got, and executed as follows, viz.

Procure a box of mahogany ABCD (Fig. 6.) of about a foot and a half long, and made in the ſhape of a truncated pyramid, whoſe baſe BDFG is eight inches wide, and ſix inches high. Fix to the other end of it a tube of four or five inches long, and which you can draw out from the box more or leſs. Let the inſide of the box be made quite black, and let the box be placed on a ſtand of wood H, and on which it may be elevated or depreſſed by the hinge I. Take a ſmall frame of wood ABCD, (Fig. 7.) and divide it at every inch by lines of black ſilk drawn acroſs it, forming 48 equal parts; divide theſe into ſtill ſmaller equal parts, by lines of finer ſilk, and fix this frame at the end BD, as the baſe of the pyramid.

Provide a drawing paper, divided into the ſame number of parts as is the frame, by lines lightly drawn in chalk. It is not material of what ſize theſe diviſions are; that will depend entirely on the ſize you propoſe to draw the objects by this inſtrument.

Place this inſtrument oppoſite a landſcape, or any other object that you want to draw, and fix the leg firmly on, or in the ground, that it may not ſhake; then turning it to the ſide you chooſe, raiſe or incline it, and put the tube farther in or out, till you have gained an advantageous view of the object you intend to draw.

Place yourſelf by the inſtrument, which you have adjuſted to the height of your eye, and looking through the tube, carefully obſerve all that is contained in each diviſion of the frame, tranſpoſe it to the correſponding diviſion in your paper; and if you have the leaſt knowledge in painting, or even drawing, you will make a very pleaſing picture, and one in which all the objects will appear in the moſt exact proportion.

VII. The CONSTRUCTION and USE of a DOUBLE PARALLEL RULER in Perſpective, Architecture, &c.

AS no inſtrument is of ſo great and general uſe in all kinds of drawing, in architecure, perſpective, &c. as the parallel rule, I ſhall here give the conſtruction and deſcription of one beſt calculated for perſpective, drawing and deſigns in particular, and may be properly called a double parallel ruler.

The conſtruction of this ruler is immediately derived from the mechaniſm and contexture of the two quadrangles IKLM, and LMNO; and is in reality nothing elſe, excepting that the parts or bars IK and NO are of double the length continued out at the ends K and N; and the middle or ſmall bar LM made longer at the end M. All this is evident in the form of the ruler itſelf (Fig. 9.) a little opened, where the ſame letters denote the ſame parts in both Fig. 8. and 9.

The deſign of this parallel ruler is to command the whole extent of the perſpective table, without being moved, that is, one ſide of the ſaid rule IK (Fig. 9.) being fixed by two ſteady pins to the lower part of the frame EF, the other bar or ſide of the ruler ON is capable of being opened parallel-wiſe over the whole table, or quite to the top GH; or from the ſide FG to the ſide EH, at one opening.

THIS opening of the ruler is not only parallel, but perpendicular likewiſe; or the ends of the moveable bar are always perpendicular over the ends of the fixed one. This evidently follows from its being a double ruler, or conſiſting of two ſingle ones connected, viz. IKLM and LMNO (Fig. 8.) For as much as the end L over-runs the end K in the firſt, juſt ſo much muſt the end O go out beyond M in the ſecond; conſequently the points O and N will be vertical to I and K in the whole operation.

FROM this conſtruction of the double parallel and perpendicular ruler, it appears, that the extent of the opening will be equal to twice the length of the ſide pieces lM or MO (Fig. 8.) that is, nearly the length of the ruler itſelf, as repreſented in Fig. 9. Hence it is plain, that by this contrivance all lines in the perſpective drawing on the table which are parallel to the ſides, may be examined, continued, and completed, without the leaſt trouble, and with the greateſt certainty and truth. Alſo all other lines in the drawing are commanded at one opening and placing the ruler; and ſo all the conſtituent lines being drawn, the whole perſpective or landſcape is finiſhed and completed, in much leſs time, and with greater eaſe and truth, than it could be by common parallel rulers.

The principles, therefore, and rationale of the perſpective table and double parallel ruler are the ſame in each; and it would be more prolix than difficult to ſhew how very uſeful ſuch a plain drawing table and parallel ruler would be in all branches of practical mathematics, and particularly in drawing the various orders of architecture, edifices, and deſigns of all kinds.

I SHALL conclude with one obſervation more, and that is, that ſince the end O of the moveable bar NO always ſlides along the ſide EH of the table, it is evident, that if a line of inches, (or any other equal parts) were placed upon the ſide EH, and a proper nonius upon the end of the ruler at O, that then parallel lines might be drawn at any given diſtance from each other, to the 100th or 400th part of an inch, if required. Hence there will be no need to expatiate on the extreme utility of the ruler thus improved in drawing the *parallel lines at their requiſite diſtances*, which conſtitute the ſeveral members and ornaments of pillars, pilaſters, and their pedeſtals, &c. in every order, and according to the meaſures propoſed by the moſt celebrated *ancient or modern* architects.

VIII. The PRINCIPLES of SPHERICAL PERSPECTIVE demonſtrated, and applied univerſally to the Geographical and Aſtronomical Projections of the Sphere in Plano, for the Conſtruction of Maps, Planiſpheres, Analemna, &c.

THE principles of ſpherical perſpective are next to be explained, and from thence it will appear, that they are the true baſis of all the variety of projections of the ſphere in plano, that are ſo common in aſtronomical and geographical treatiſes; and not only ſo, but it will be manifeſt, that the doctrine of ſpherical projections, which, treated in the common way, is very intricate and confined, will, by the rules of perſpective, be rendered facile and univerſal.

To this end it will be neceſſary to conſider the conſtruction of a figure conſiſting of three planes, (as Fig. 10.) of which one is the perſpective plane, or plane of projection AB; the ſecond is CD, in a vertical poſition to the horizon, and is therefore called the vertical plane; the third EF, which paſſes through the eye at I, and being parallel to the horizon, is called the horizontal plane. Theſe planes are all ſuppoſed to interſect each other at right angles; whence the interſection MN is called the horizontal line, and OP the vertical line. And laſtly, the line

IL is called the *principal ray*, as coinciding with the axis of the eye.

Let H be a given point, whoſe perſpective, or ſeat *(h)* is to be determined upon the perſpective plane AB; from point H let fall the perpendicular HK to the vertical plane, and HG to the horizontal plane; and draw the lines IK, IH, IG, which will paſs through the plane of projection in the points *k*, *h*, *g*. Then the poſition of the point *h*, in the plane of projection AB, with reſpect to the vertical and horizontal lines OP and MN, is thus determined.

The planes being all given in the poſition, their interſecting lines MN, OP, are given in poſition on the plane of projection. Alſo the poſition of the line IL is given; and becauſe the poſition of the object H is given, its diſtance HK from the vertical plane, and HG from the horizontal plane, are known; alſo its diſtance lL from the perſpective plane, as likewiſe that of the eye I, l, are both known. Then in the ſimilar triangles ILK, and Ilk, we have $IL : Il :: IK : Ik$ and the ſimilar triangles IKH, Ikh give $IK\ Ik : HK : hk$; conſequently we have from both analogies $IL\ (= Il \times Ll : Il :: HK : hk$; which in words is

GENERAL RULE FIRST.

AS the ſum of the diſtances of the eye and the object from the plane of projection is to the diſtance of the eye from that plane; ſo is the diſtance of object from the vertical plane to the diſtance of its ſeat from the vertical line.

And in like manner, we have IL : I*l* : HG : *hg*; which gives in words,

GENERAL RULE SECOND.

AS the ſum of the diſtances of the eye and object from the plane of projection is to the diſtance between the eye and this plane; ſo is the diſtance of the object from the horizontal plane to the diſtance of its image from the horizontal line. It is manifeſt, that if the object be placed between the eye and the perſpective plane, theſe proportions will be the ſame, only uſing the word *difference* inſtead of *ſum* in the firſt term of each.

To apply what has been premiſed, it is evident, that in projections of the ſphere, the above mentioned three planes may be applied, but they will be of a circular form; and then, by theſe rules, any point upon the ſurface of the globe or ſphere will have its ſeat aſcertained in the plane of projection, for any poſition or diſtance of the eye whatever; but all diſtances in this caſe muſt be eſtimated in parts of the radius to a table of ſines, as will be eaſy to underſtand by the following examples.

Let it be required to find the perſpective or ſeat of any point upon the ſurface of the globe, projected upon the plane of the equator, as ſeen by the eye at a given diſtance in the axis of the globe produced.

The three planes are in this example (1.) AB, the *plane of projection*, or that of the equator. (2.) CD, the vertical plane, which is that of the *equinoctial colure*. (3.) EF, the horizontal plane, which is that of the *ſolſtitial colure*; for theſe three planes are all at right angles to each other.

The place of the ſtar upon tbe globe being given by a table of right aſcenſions and declinations, we ſhall have (by the *doctrine of the ſphere)* this analogy.

As the radius is to the ſine of right aſcenſion, ſo is the co-ſine of declination to the ſtar's diſtance from the vertical plane ; and then again, As radius to the co-ſine of right aſcenſion, ſo is the co-ſine of declination to the ſtar's diſtance from the horizontal plane.

The diſtance of the ſtar from the plane of projection is the ſine of its declination, and the diſtance of the eye from the ſaid plane is to be expreſſed by ſome multiple of radius, as nR; the value of (n) being taken at pleaſure, and R=10000=radius.

Thus let the place of *Aldebaran*, or the Bull's Eye, be required in a planiſphere upon the plane of the equator; the declination of the ſtar is 16 degrees north; the ſine of which is 2756, therefore that is the diſtance of Aldebaran from the plane of projection.

THE right aſcenſion of the ſtar is 65° 30′ therefore

ſay as radius - 10

To the ſine of right aſcenſion - - 65° 30′ 9,959,023

So is the co-ſine of declination - 74° 0′ 9,982,842

To the ſtars diſtance from the vertical plane 8747—9,941,865

Then again, as radius 10

To the co-ſine of right aſcenſion - 24° 30′ 9,617,727

So is the co-ſine of declination - 74° 0′ 9, 982, 842

To the diſtance of the ſtar from the horizontal plane - 3986, 2 9,600,569

HAVING the diſtance of the ſtar from the three planes, and the diſtance of the eye $n\mathrm{R}=10000n$, you ſay (by general rule 1.) as $1000n+2756 : 10000n : : 8747 : \frac{n\times 87470000}{10000n+2756}$ = diſtance of Aldebaran from the vertical line in the planiſphere.

Then (by General Rule 2.) ſay,

AS $10000n+2756 : 10000n : : 3986, 2 ; \frac{n+39862000}{10000+2756} =$ the diſtance of the ſtar from the horizontal line in the projection.

THUS you have the place or ſeat of the ſtar in the planiſphere for any diſtance of the eye whatſoever.

IF we put $n = 1$, then is the eye upon the ſurface of the globe, viz. in the South Pole of the world, and the diſtance of Aldebaran from the vertical line will be $\frac{87470000}{12756}=6857,3$ of ſuch parts as the radius of the planiſphere contains 10000.

Again the diſtance of the ſtar from the horizontal line is, in this caſe, $\frac{39862000}{12756}=3125$. And thus the places of all the ſtars are put down in a celeſtial planiſphere, according to the common *ſtereographic projection* upon the plane of the equator.

The ſtars may in the ſame manner be projected upon the plane of the ecliptic; and thus from the principles of perſpective, we ſee the reaſon and foundation of thoſe two excellent celeſtial projections of the conſtellations upon the planes of the ecliptic and equator, firſt publiſhed ſome years ago by Mr. Senex.

If the ſurface of the terreſtrial globe were to be thus projected upon the plane of the equator, the places muſt be put down by the ſame rules; but inſtead of declination, you take the latitude of the place; and for right aſcenſion you take the longitude from the firſt meridian, or vertical plane. And by this means you will have the old Ptolomaic map of the world, in the centre of which is the North Pole; the meridians are all right lines, and the parallels of longitude are all concentric circles.

The common maps of the world are projections upon the plane of the firſt meridian, taken at pleaſure; the vertical circle is the plane of the meridian at 90 degrees from it, and the horizontal circle is the plane of the equator; and in the interſection of the two laſt, the eye is placed at the diſtance *n* R, as before.

Thus, for example, let the meridian of London be the plane of projection, and let it be required to find the ſeat of the city of Conſtantinople in the map; the latitude of this city is 41°, and the longitude eaſt 29°. Then the diſtance from the horizontal plane is the ſine of latitude; and ſince the difference of longitude between this city and the vertical plane is 61°, you ſay,

As radius to the ſine of the difference of longitude, ſo is the co-ſine of the latitude to the diſtance of Conſtantinople from the vertical plane.

And again, as radius to the co-ſine of the difference of longitude, ſo is the co-ſine of the latitude to the diſtance of the city from the plane of projection.

If we put $n=1$ as before, then the eye is upon the ſurface of the globe, viz. in the pole of oppoſite the hemiſphere; and the map becomes that of the vulgar ſtereographic projection.

If the place be in the equator, its diſtance from the horizontal plane is=O; and its diſtance from the vertical plane is the ſine of the difference of longitude, which put $=s$, and the co-ſine thereof = C, then (by Rule I.) we have $nR + c : nR : : s : D =$ diſtance from the vertical line, in the horizontal line of the map, and when $n=1$, it will be $\frac{Rs}{R+c}=D$. Let $s=$ſine of 10° then D=8749. But if $s=$ſine of 80°, then D=8391,4, which taken from radius = 10000, leaves

1608,6; therefore the width of 10° of the equator in the middle of a map will be to the width of 10°, at the ſide, as 875 to 1608½, conſequently it proves how much ſuch maps muſt differ from the truth of the globes. Let the radius or half the horizontal line be divided into a number of equal parts, denoted by *(p)* and let *m* denote any number of them; then will $\frac{m\ R}{p}=\frac{n\ R\ s}{nR+c}$ whence we have $n=\frac{m\ c}{spmR}$.

HENCE, if $p=90$, and $m=10$, 20, 30, &c. ſucceſſively, you will have the diſtance the eye, viz. nR given for projecting all the meridians at an equal diſtance upon the map, and which therefore may be entitled the globular projection, or map of the world.

FOR example, ſuppoſe $m=10$, and $p=90$ then $m : p :: 1 : 9$, therefore $n=\frac{c}{sp-R}=$ 1,7496; and therefore $nR=17496$, the diſtance of the eye for projecting the meridian of 10 degrees, and ſo for the reſt.

If we ſuppoſe the eye at an infinite diſtance, then all the meridians are projected (by the parallel viſual rays) into ellipſes; and the diſtance of any ſtar from the horizontal line will be the ſine of its declination; and radius will be to the ſine of right aſcenſion, as the co-ſine thereof is to the diſtance of the ſtar from the vertical line in the projection.

This projection is of little uſe in geography, but of very great uſe in aſtronomical ſciences, and is known by the names of analemma and artheographical projections of the ſphere.

FINIS.

Fig. 8.

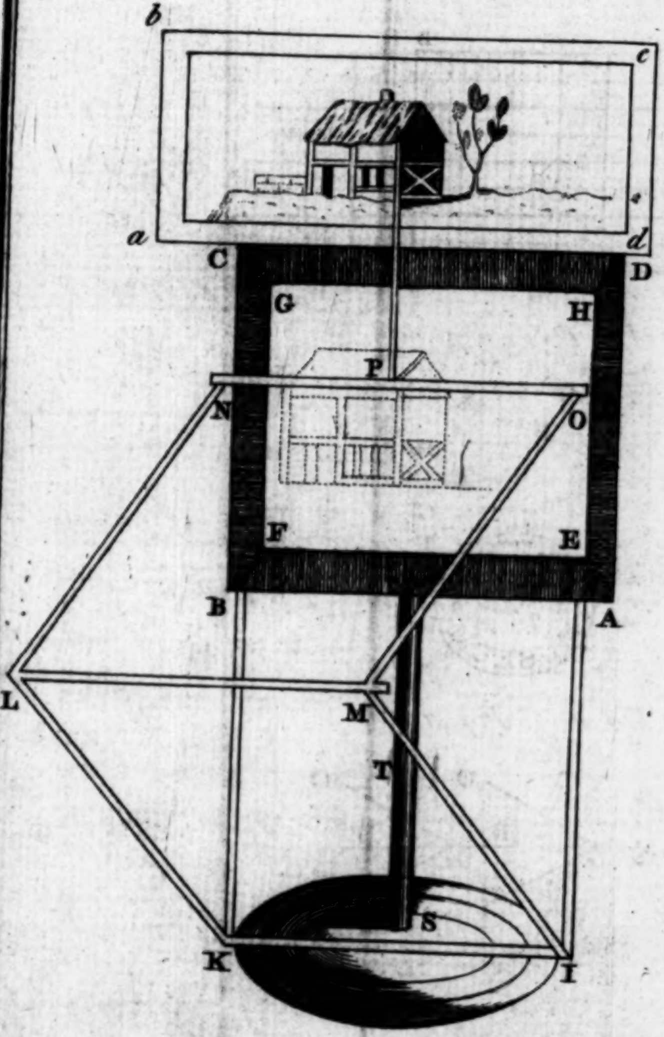
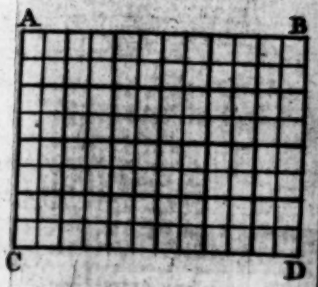


Fig. 7.



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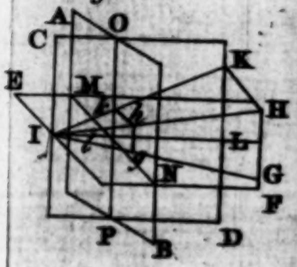

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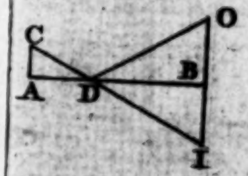

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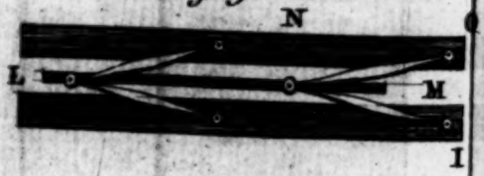

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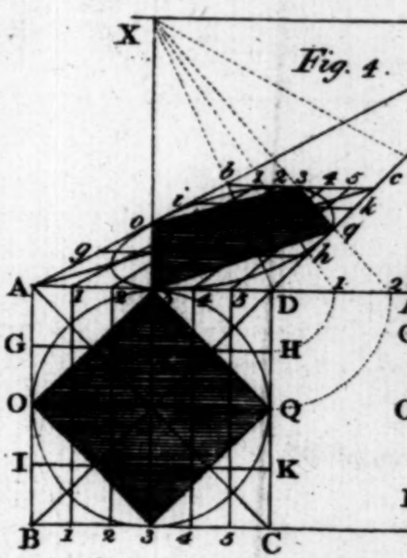
Fig. 4.



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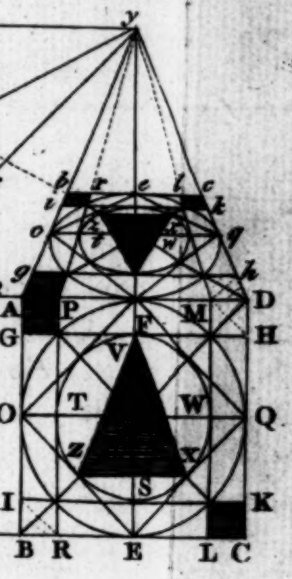

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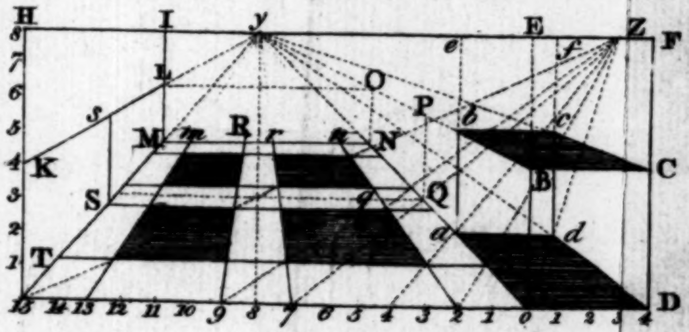

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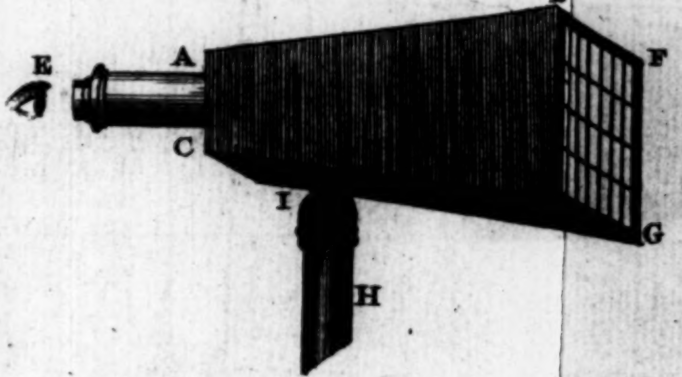

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Fig. 1.

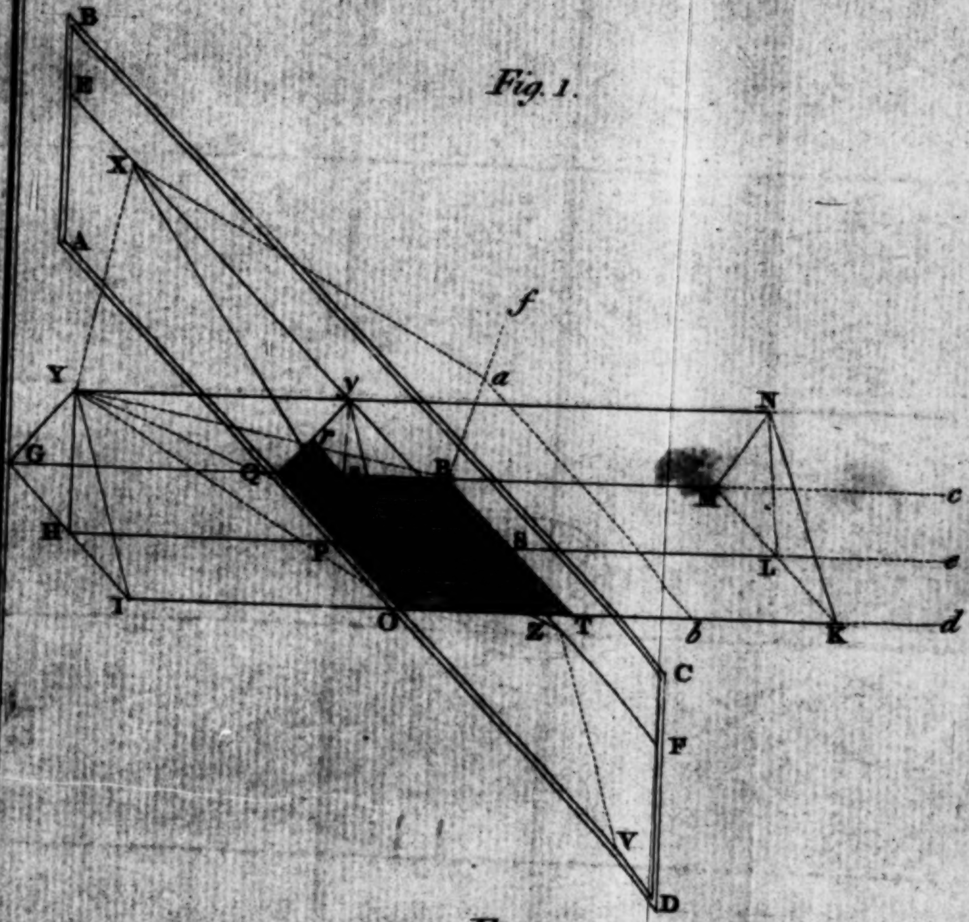
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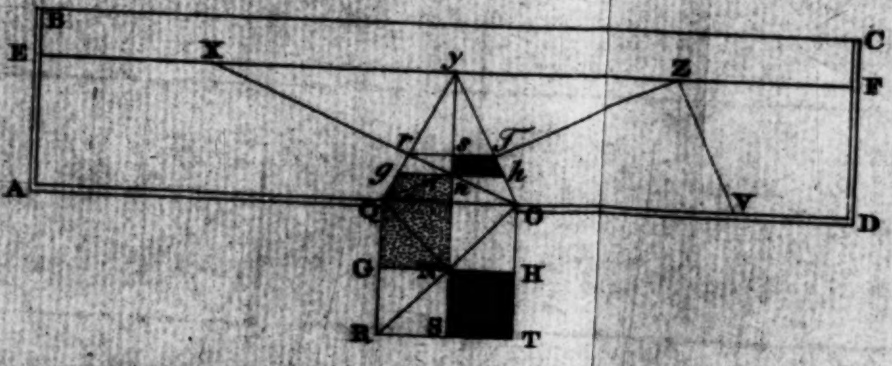
Fig. 2.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